UNIT - 2

Set Theory

A set theory is any well defined collection of objects is called as elements & members of the sets It is denoted by { } the letters A, B, C.

Eg:- collection of students in the class represented.

There are two ways

1. Roaster form (or) Tabular form
2. Set builder form (or) Rule method

Eg:- list of even no's b/w 1 to 20 by using roaster form by set builder form.

Roaster form :- {2, 4, 6, 8, 10, 12, 14, 16, 18}

Set builder form :- {x/x is an even no's x<2

Equality of two sets :- Two sets are equal if and only if they have same elements.

$A \subseteq B$ & $B \subseteq A \Rightarrow A = B$

Eg:- $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$

subset :- The set A is said to be subset of B if and only if every element of A is also a element of set B

It is denoted by $A \subseteq B$.

Properties of subset :

* Every set is a subset of B
* two sets A & B are equal if and if A is subset of B ~~is a sub set of everyset {} ϕ~~ and B is subset of A
* The null set is a sub set of everyset {} ϕ
* for any set A is C if A is subset of B, B is subset of A, A is subset of C.

~~Null Set~~ 3)

Finite Set :-

A set with finite numbers of elements in it, is called a finite set

Ex :- The set vowels in English alphabets

Infinite set :-

An infinite set is a set which contains number of elements (or) uncountable.

Ex :- A = a set of integers = {0, 1, 2, 3 ... }

Null Set :-

A set which contains no elements at all is called the Null set also known as Empty set. It is denoted by ϕ

~~Eg~~ Singleton Set :-
A set which has only one element is called a singleton set.
for eg. $S = \{a\}$ is a singleton set

Power set :- For a set A a collection of all subsets of A is called the power set of A and it is denoted by P(A) (or)

The set of all subsets of a set A is called powerset

eg :- $A = \{1, 2\}$

1) $P(A) = \{1, \{1\}, \{2\}, \{3\}\}$

2) The powerset of $A = \{0, 1, 2\}$

$P(A) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$

$2^3 = 8$

Super set :- If A is a subset of B, then B is called the superset of A and is written as $B \supseteq A$ which is read as B is a superset of A

Proper subset
Any subset A is said to be proper subset of another set B if A is a subset of B, but there is atleast one element of B which does not belong to A i.e., if $A \subseteq B$ but $A \neq B$ it is

written as $A \subset B$. It is also called proper inclusion (6)

Eg:- If $A = \{1,5\}$, $B = \{1,5,6\}$; $C = \{1,6,5\}$

Then A & B are both subsets of C; but A is a proper subset of C, where as B is not a proper subset of C since $B = C$. A proper inclusion is not reflexive, but it is transitive

i.e. $(A \subset B) \cap (B \subset B) \Rightarrow (A \subset C)$

Equal sets :-

Two sets A and B are said to be equal if and only if every element of A is an element of B and consequently every element of B is an element of A, that $A \subseteq B$ and $B \subseteq A$ and it is written as $A = B$

Operations on sets

Union :- The union of two sets A and B, denoted by $A \cup B$, pronounced as 'A union B' is the set of all elements which belongs to A or to B or to both, that is, $A \cup B = \{x : x \in A \text{ or } x \in B\} =$

$\{x \mid x \in A \vee x \in B\}$

intersection :-

The intersection of two sets A and B, denoted by $A \cap B$, pronounced by A intersection B is the set of elements which belong to both A and B.

Complements

Let U be the Universal set and A be any subset of U. The absolute complement of A or, simply, complement of A, denoted by A' or A^c is the set of elements which belong to U but which do not belong to B.

Symmetric difference

The symmetric difference of two sets A and B, denoted by $A \Delta B$ or $A \oplus B$ is the set of elements that belong to A or to B, but not to both A & B. It is also called the Boolean sum of two sets.

Algebra of Sets

The laws satisfying by the set operations form the algebraic laws of set operations.

Cartesion product of two sets

Let A & B are two non-empty sets then the cartesion product A & B is denoted by $A \times B$ is denoted as $A \times B = \{(a, b) / a \in A, b \in B\}$

Eg:- If $A = \{1, 2, 3\}$ $B = \{a, b\}$ then

$A \times B$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$n(A) = m \qquad n(A \times B) = m \times n$

$n(B) = n$

1. Idempotent Laws

a) $A \cup A = A$ b) $A \cap B = A$

2. Associative laws

a) $(A \cup B) \cup C = A \cup (B \cup C)$ b) $(A \cap B) \cap C = A \cap (B \cap C)$

3. Commutative laws

a) $A \cup B = B \cup A$ b) $A \cap B = B \cap A$

4. Distributive laws

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Identify laws.

a) $A \cup \phi = A$ (b) $A \cap U = A$

6 a) $A \cup U = U$ (b) $A \cap \phi = \phi$

7 Involution law

$(A')' = A$

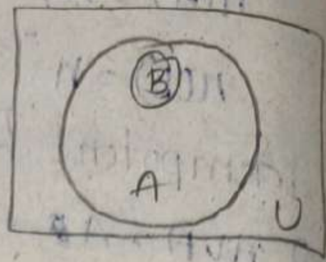
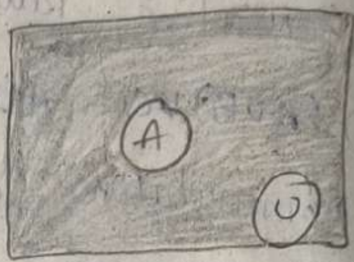
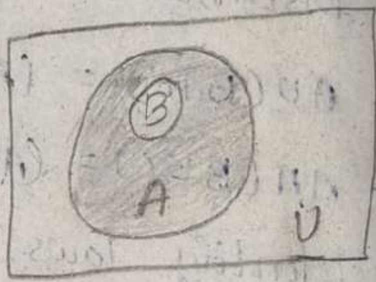
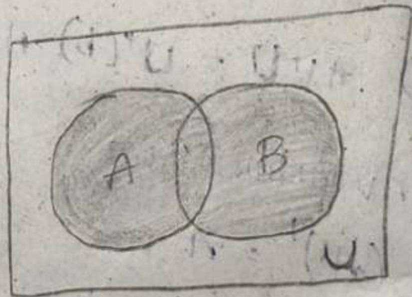
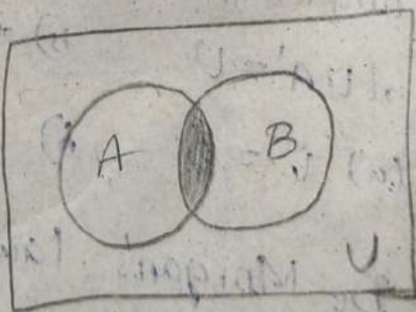
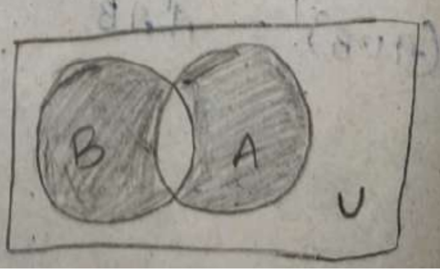
8. Complement laws.

a) $A \cup A' = U$ b) $A \cap A' = \phi$

9. (a) $U' = \phi$ b) $\phi' = U$

10. De Morgan's law

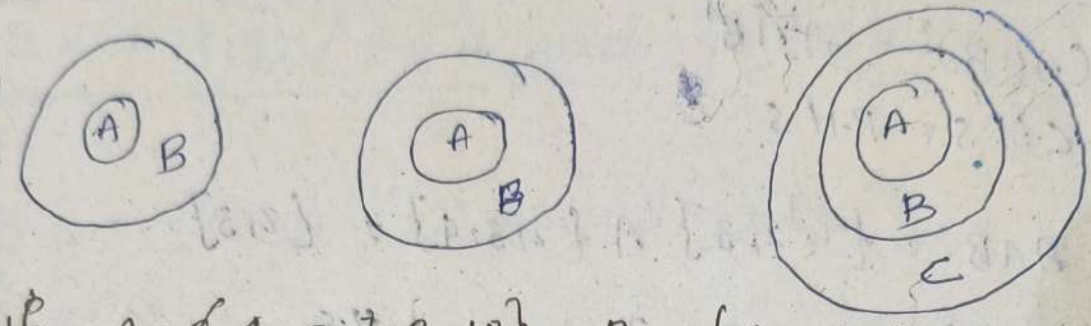
a) $(A \cup B)' = A' \cap B'$ (a) $(A \cap B)' = A' \cap B'$

Venn diagram	Symbol	Venn Diagram
Set operations		
Set B is a proper subset of A	$B \subset A$	
The Complement of set A	A'	
The difference of sets A and B	$A-B$	
The Union of Sets A and B	$A \cup B$	
the intersection of sets A and B	$A \cap B$	
The Symmetric difference of sets A and B	$A \Delta B$	

Ex (1) Use Venn diagram to illustrate the relationship (8)

(i) $A \subset B$ (ii) $A \subseteq B$ (iii) $A \subset B$ and $A \subset C$

Sol:- (i)



Ex:- If $A = \{4, 5, 7, 8, 10\}$ $B = \{4, 5, 9\}$ and $C = \{1, 4, 6, 9\}$ then verify that i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol:- $B \cup C = \{4, 5, 9\} \cup \{1, 4, 6, 9\} = \{1, 4, 5, 6, 9\}$

$A \cap (B \cup C) = \{4, 5, 7, 8, 10\} \cap \{1, 4, 5, 6, 9\} = \{4, 5\}$

$A \cap B = \{4, 5, 7, 8, 10\} \cap \{4, 5, 9\} = \{4, 5\}$

$A \cap C = \{4, 5, 7, 8, 10\} \cap \{1, 4, 6, 9\} = \{4\}$

Now $(A \cap B) \cup (A \cap C) = \{4, 5\} \cup \{4\} = \{4, 5\}$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S = R.H.S //

(3) If $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$ and $U = \{1, 2, 3, 4, 5, 6\}$ verify that 1) $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

Sol:- $A \cup B = \{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$

$(A \cup B)' = \{5, 6\}$ — L.H.S

$A' \cap B' = \{4, 5, 6\} \cap \{1, 5, 6\}$

$A' \cap B' = \{5, 6\}$ — RHS.

Hence the given problem is

$(A \cup B)' = A' \cap B'$

L.H.S = R.H.S.

(ii) $A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$

$(A \cap B)' = \{1, 4, 5, 6\}$ —— L.H.S

$A' \cup B' = \{4, 5, 6\} \cup \{1, 5, 6\}$

$= \{1, 4, 5, 6\}$ — R.H.S

Hence the given problem is $(A \cap B)' = A' \cup B'$

L.H.S = R.H.S.

(4) Let A & B be two sets show that

1) $(A \cap B) \subseteq A$ 2) $A \cap (B - A) = \phi$

Sol:- 1) Let $x \in A \cap B = x \in A$ and $x \in B$

$\Rightarrow x \in A$ [∵ By using Intersection]

$\Rightarrow (A \cap B) \subseteq A$

(ii) $x \in A \cap (B - A) = x \in A$ and $x \in (B - A)$ [∵ $A - B = x \in A$ & $x \notin B$]

$= x \in A$ and $x \in B$ and $x \notin A$ [∵ $x \notin A = x \in A'$]

$= x \in A$ and $x \in A'$ and $x \in B$

$= x \in (A \cap A') \cap B$ [∵ $A \cap A' = \phi$]

$= x \in (\phi \cap B)$ [∵ $A \cap \phi = \phi$]

$= x \in \phi$ [$(\phi \cap B) = \phi$] By using Intersection

$\Rightarrow x = \phi$ $A \cap (B - A) = \phi$

(5) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (10)

Proof:- Let us consider

$$\boxed{A \times B = \{(x,y) / x \in A \text{ and } y \in B\}}$$

Now

$A \times (B \cap C) = \{(x,y) / x \in A \text{ and } y \in (B \cap C)\}$

$= \{(x,y) / x \in A \text{ and } (y \in B \text{ and } y \in C)\}$

$= \{(x,y) / (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)\}$

$= \{(x,y) / (x,y) \in A \times B \text{ and } (x,y) \in A \times C\}$

Based given problem

$A \times (B \cap C) = \{(x,y) / (x,y) \in (A \times B) \cap (A \times C)\}$

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(6) If $A = \{1,2,3\}$ $B = \{4,5\}$ $C = \{1,2,3,4,5\}$

find i) $A \times B$ ii) $C \times B$ iii) $B \times B$ also prove that $(C \times B) - (A \times B) = B \times B$

Soln:- (i) $A \times B = \{(1,2,3) \times (4,5)\}$

$= \{(1,4), (1,5) (2,4) (2,5), (3,4) (3,5)\}$

(ii) $C \times B = \{(1,2,3,4,5) \times (4,5)\}$

$= \{(1,4), (1,3) (1,4) (2,5) (3,4) (3,5)$

$(4,4) (4,5) (5,5)\}$

$B \times B = \{(4,5) \times (4,5)\}$

$= \{(4,4)(4,5)(5,4)(5,5)\}$

$((C \times B) - A \times B = B \times B$

$= \{(1,4)(1,5)(2,4)(2,5)(3,4)(3,5)\}$

$- \{(1,4)(1,5)(2,4)(2,5)(3,4)(3,5)(4,4)$
$(4,5)(5,4)(5,5)\}$

$= \{(4,4)(4,5)(5,4)(5,5)\}$

$((C \times B) - (A \times B) = \{(4,4)(4,5)(5,4)(5,5)\}$

Hence proved //

(7) Let A, B, C be an arbitary sets

Show that $(A-B)-C = (A-C) - (B-C)$

Sol :- $\boxed{A-B = \{x/x \in A \text{ and } x \notin B\}}$

$(A-B)-C = \{x/x \in A \text{ and } x \notin B \cap x \notin C\}$

$(A-C) = \{x/x \in A \cap x \notin C\}$

$B-C = \{x/x \in B \cap x \notin C\}$

$x \in B = x \notin B$ Based on orbitary form

$(A-C)-(B-C) = \{x/x \in A \cap x \notin B\} \cap \{x/x \in B \cap x \notin C\}$

$= x/x \in A \cap x \notin B \cap x \notin C$ /

Hence proved //

28/03

Collection of sets :- If the elements of a set themselves, then such a set is said to be a 'collection of sets' or 'class of sets' or 'family of sets'.

If we wish to consider some of sets in a gimen class of sets, then we speak of a subclass or sub collection.

Arbitary union of sets :- Let $\{A\}_{i \in I}$ be an indexed family of sets, then the arbitary union of sets A_i, to be denoted be $\bigcup_{i=1} A_i$ is the set of elements that belong to at least one A_i More compactly:

$$\bigcup_{i=1} A_i = \{x : x \in A_i \text{ for some } i \in I\}$$

Arbitary Intersection of sets :- The arbitary intersection of the sets A_i, to be denoted by $\bigcap_{i \in I} A_i$ is the set of elements that belong to all A_i. More compactly:

$$\bigcap_{i \in I} A_i = \{x : x \in A_i \text{ for all } i \in I\}$$

An indexed family of sets $\{A_i\}_{i \in I}$ is said to be disjoint if $\bigcap_{i \in I} A_i \neq \phi$, and family is said to be pairwise disjoint if $A_i \cap A_j = \phi$ whenever $i \neq j$

Partition of a set :- A partition of a set A is collection of non-empty subsets $A_1, A_2, \ldots A_n$, called blocks, such that each element of A is in exactly one of the blocks. That is

1) A is the union of all the subsets ~~$A_i \cap A_j$~~ $= \emptyset$ ~~for~~ i $A_1 \cup A_2 \cup \ldots \cup A_n = A$ and.

2) The subsets are pairwise disjoint, $A_i \cap A_j = \emptyset$ for $i \neq j$.

Multiset :- Multisets are sets where an element can occur as a member more than once for example, $A = \{a, a, a, b, b, c\}$

$B = \{a, a, a, a, b, b, b, c, c\}$ are multisets

The multisets A and B can also be written as $A = \{3.a, 2.b, 1.c\}$ and $B = \{4.a, 3.b, 2.c\}$

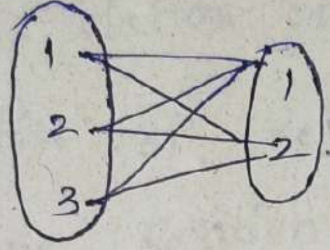
Operations on multisets :- Let A and B be multisets. The union of A and B, denoted by $A \cup B$, is the multisets where the multiplicity of an element is the maximum of its multiplicities in A and B.

Relations :-

If A & B are two non-empty set the subset of cartesion product is called as a Relation

$$A \times B = \{ (x, y) , x \in A \text{ and } y \in B \}$$

Eg :-



Domain = {1, 2, 3}

Range = {1, 2}

$$A \times B = \{ (1,1) \ (1,2) \ (2,1) \ (2,2) \ (3,1) \ (3,2) \}$$

Domain :- Set of all the 1st element in the ordered pair is called as domain

The set $\{ a \in A , (a,b) \in R$ for some $b \in B \}$ is called the domain R and denoted by Dom (R)

Range :- Set of all the 2nd element in the ordered pairs is called as Range.

The set $\{ b \in A , (a, b) \in R$ for some $a \in A \}$ is called the Range R and Denoted by Ran (R)

Types of Relation

- Inverse Relation
- Identity Relation
- n-ary Relation

Inverse Relation :- let R any relation from A to B. The inversed of R denoted by R^{-1} is the Relation from B to A which is consists of those ordered pairs.

$R = \{ (a,b) . (a,b) \in R \}$

$R^{-1} = \{ (b,a) : (a,b) \in R \}$

Ex :- $R = \{ (1,2) (2,3) \}$

$R^{-1} = \{ (2,1) (3,2) \}$

Identity Relation :-

A Relation R in a set A is said to be Identity generally denoted by

$I_A = \{ (x,x) : x \in A \}$

Ex :- let $A = \{ (1,2,3) \}$

$I_A = \{ (1,1) (2,2) (3,3) \}$ Identity Relation in A

n-ary Relation :-

Let $\{ A_1, A_2, A_3 \cdots A_n \}$ be a finite collection of sets. A subsets of R of $\{ A_1 \times A_2 \times A_3 \times \cdots \times A_n \}$ is called as n-ary Relation

1) If $R = \phi$ then R is called void (or) empty Relation

2) If $R = A_1 \times A_2 \times A_3 \cdots \times A_n$ then R is called the universal Relation

3) If $A_i = A$ for i, then R is called on n-ary Relation on A / Identify Relation

4) For $n = 1, 2, 3 \ldots$ R is called as unary, binary, ternary relation respectively.

properties of Relation :-

1) Reflexive Relation :- A Relation R on a set A is Reflexive if $a R a$ for every $a \in A$, that $(a,a) \in R$, for every $a \in A$ this simply each element a of A is related to itself.

Ex:- $A = \{(1,2,3)\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ to be a relation of $A = \{(1,2,3)\}$ then R_1 is Reflexive since every $a \in A$.

2) Irreflexive Relation :- A Relation R on set A is irreflexive if, for every $a \in A$, $(a,a) \notin R$

Ex:- The Relation $A = \{(1,2,3)\}$

$R_1 = \{(1,2)\ (1,3)\ (2,1)\ (2,3)\}$ is irreflexive relation since $(x,x) \notin R_1$ for every $x \in R_1$

(3) Symmetric Relation :- A Relation R on set A is symmetric if whenever $(a,b) \in R$ the (17)
$(b,a) \in R$ if $aRb = bRa$. This means If any one element is Related to any other element then the second element is Related to 1st element

(a) $A = \{1, 2, 3\}$

$R_1 = \{(1,2)\ (2,1)\ (3,1)\ (1,3)\}$

(4) ~~Asymmetric~~ Relation :- Let A Relation R on set A is asymmetric if whenever $(a,b) \in R$ then $(b,a) \notin R$ for $a \neq b$ i.e. if

$aRb = a \not R a$

Ex $A = \{(1, 2, 3)\}$

$R_1 = \{(1,1)\ (1,2)\ (2,3)\ (3,1)\}$

It is a a symmetric Relation.

(5) Antisymmetric Relation :-

A Relation R on a set A is anti symmetric if aRb and bRa $a = b$ for all $a, b \in A$ that is if $(a,b) \in R$ and $(b,a) \in R$ implies $a = b$

Ex :- $A = \{1, 2, 3\}$

$R_1 = \{(1,2)\ (2,2)\ (2,3)\}$ is an antisymmetric relation.

⑥ Transitive Relation :- A Relation R on a set A is transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ ie aRb, bRc, aRc. This means if one element is related to second and second element is Related to third and first element is Related to third element.

property	meaning
1) Reflexivity	$(a,a) \in R$ i.e aRa for all $a \in A$
2) Irreflexivity	$(a,a) \notin R$ ie $a \not R a$ for all $a \in A$
3) Symmetric ~~Irreflexivity~~	$(a,b) \in R \Rightarrow (b,a) \in R$ ie, $aRb \Rightarrow bRa$ for all $a,b \in A$
4) Asymmetric	$(a,b) \in R \Rightarrow (b,a) \notin R$ i.e. $aRb \Rightarrow b \not R a$ for all $a,b \in A$
5) Antisymmetric	$(a,b) \in R \wedge (b,a) \in R$ i.e $a=b$ for aRb and $bRa \Rightarrow a=b$ for all $a,b \in A$
6) Transitivity	$(a,b) \in R \wedge (b,c) \in R \Rightarrow (a,c)$, aRb and bRc for all $a=c$.

Equivalence Relation :- A Relation on A set A is called Equivalence relation. (19)

(or)

A Relation satisfy Reflexivity, symmetric transivity

Anti = (1,2) = (1,2)

(a,b) (b,c) (a,c)

$\boxed{a=c}$

Problem :-

Ex :- let $x = \{1,2,3,4,5,6,7\}$ and

$R = \{(x,y) / x-y$ is divisible by $3\}$

Show that R is an Equivalence relation.

Sol :- Given $x = \{(1,2,3,4,5,6,7)\}$ and

$R = \{(x,y) / x-y$ is divisible by 3

$R = \{ (1,1) (1,4) (1,7) (2,2) (2,5) (3,3) (3,6)$
$(6,3) (6,6)$
$(4,1) (4,4) (4,7) (5,2) (5,5)$
$(7,1) (7,4) (7,7)\}$

All ordered pairs are divisible by 3

Reflexivity :- for given

$R = \{(x,x) : x \in x\}$

the given problem Reflexivities

are $R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\ (7,7)$

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Symmetric :- Relation R is symmetric $(2,5)$
$(5,2)\ (1,7)\ (7,1)\ (3,6)\ (6,3)\ (4,7)\ (7,4)$
$(1,4)\ (4,1) \in R$

Transitivity :- $(a,b) \in R,\ (b,c) \in R,\ (a,c) \in R$

for $\boxed{a=c}$

given $R = \{(3,6)\ (6,3)\}$

$(3,6 \in R,\ (6,3) \in R$

$(a,c) = (3,3) \in R$

(2) Consider the following relation on $\{1,2,3,4,5,6\}$ then $R = \{(i,j) : i-j=2\}$

Is R transitive reflexiv symmetric

Is R transitive reflexiv symmetric

Sol:- Given $A = \{1,2,3,4,5,6\}$ then $R = \{(i,j)$
$i-j=2\}$

$R = \{(4,2)\ (5,3)\ (6,4)\}$

All ordered pairs are satisfy $i-j=2$

Reflexivity :- As $(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)$
$(6,6) \notin R$

So R is not reflexive on A

Symmetric :- As $(4,2) \in R$ but $(2,4) \notin R$

So, R is not symmetric on A

Transitive :- As $(4,2) \in R$, but $\{(2,4) \& (4,4)\} \notin R$. So R is not transitive on A

3) show that congruence modulom is an equivalence relation on Integer (or) prove that the relation congreence modulo 'm' given by $R = \{(x,y) / x-y$ is divisibl by $m\}$ ore the set of positive Integeos in an equivalence relation.

Sol:- 1) Reflexivity :- Let a be an Integer

$a - a = 0$ is divisible by m

Hence given a is an Reflexivity

2) Symmetric :- Let a b are two Integers

$a - b = 0$ is divisible by m $= a = b$

$b - a = 0$ is divisble by m $= b = a$

Thus the congruence a, b is are symmetric

3) transitivity :- a, b, c are 3 Integers

$a - b = 0$ divisible by m $= a = b$

$b - a = 0$ divisible by m $= b = c$

$a - c = 0$ divisible by m $= c = a$

Hence the given prob is transitive.

Thrm :- Let R be a relation on A

1) If R is reflexive, so R^{-1} is Reflexive

2) R is symmetric if and only if $R = R^{-1}$

3) R is antisymmetric if are only if $R \cap R^{-1}$ $\subseteq I_A$

proof :- i) Suppose R is Reflexive Then $(a\,a) \in R$ for all $a \in A$

so $(a\,a) \in R^{-1}$ for all $a \in A$. Therefore R^{-1} is Reflexive

ii) suppose R is symmetric let $(a, b) \in R$ the $(b, a) \in R$ Here R is symmetric and $(a, b) \in R$ & $(b, a) \in R^{-1}$. Here R^{-1} is symmetric of $R = R^{-1}$.

(iii) suppose R is an antisymmetric, let (a, b) $\in R \cap R^{-1}$ $(b, a) \in R \cap R^{-1}$ and $a = b$ is an antisymmetric Here every element of the form $R \cap R^{-1}$ the form of $(a\,a) \in A$ Here

$R \cap R^{-1} \subseteq I_A$.

(4) Suppose R and S are Relation on a Set A. Prove that

1) If R and S are Reflexive, then $R \cup S$

$R \cap S$ are reflexive

ii) If R and S are symmetric, then $R \cup S$ (23)

& $R \cap S$ are symmetric

iii) If R and S are transitive, then $R \cap S$

is tranitive (or)

If Relation R and S are Reflexive, symmetric & transive, show that $R \cap S$ is also reflexive, symmetric & transitive.

proof:-

(i) suppose R and S are Reflexive, then $(a,a) \in R$ and $(a,a) \in R \cap S$. Hence

$R \cup S$ & $R \cap S$ are Reflexive

ii) Suppose R and S are symmetric let $(a,b) \in R \cap S$ and $(b,a) \in R \cap S$ then $(a,b) \in R \cup S$ & $(b,a) \in (R \cup S)$

Hence $R \cap S$ & $R \cup S$ are symmetric.

(iii) transitive :- $(a,b) \in R$, and $(b,c) \in R$

is a $(a,c) \in R$ $\in (R \cap S$

Here given problem $(a,b) \in R \cap S$ is a (b,c)

thr and $(a,c) \in R \cap S$ is a transitive.

$(a,b) \in R \cap S$ & $(b,c) \in R \cap S$

and $(a,c) \in R \cap S$ Hence, the thrm is transitive//.

Partial order Relation (Partial ordering) :-

A Relation R on set S is called a partial order Relation if and only if R is Reflexive, antisymmetric and transitive.

* Reflexive aRa for all $a \in S$
* Antisymmetric aRb and $bRa \Rightarrow a=b$
* Transitive aRb and $bRc \Rightarrow aRc$

for example the greater or Equal $(\geq)$ relation is a partial ordering on $\mathbb{Z}$ the set of Integers.

Reflexive :- Since $a \geq a$ for every Integer a $\geq$ is reflexive

Antisymmetric :- Since $a \geq b$ & $b \geq a$, $a=b$ $\geq$ is antisymmetric

Transitive :- Since $a \geq b$ & $b \geq c$, $a \geq c$ is transitive.

Graph of a Relation :-

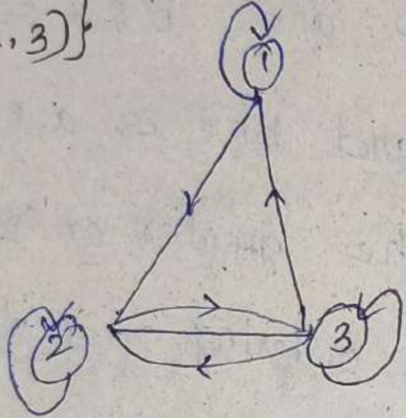
Let A & B are two finite sets and R is a Relation from A to B for graphical representation of a Relation on set. each element of the set is Represented by a point. These points are called as nodes (or) vertices.

Ex(1) :- Draw the directed graph that Represents the relation

$R = \{(1,1)\ (2,2)\ (1,2)\ (2,3)\ (3,2)\ (3,1)\ (3,3)\}$

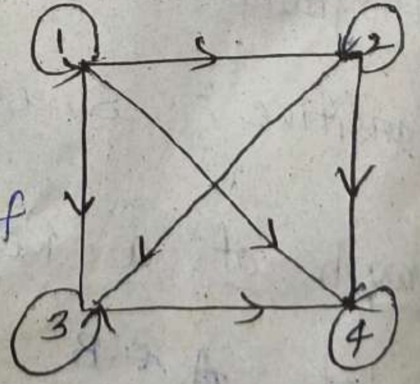
$X = \{(1,2,3)\}$

Sol:-



Each of these pair corresponds to an edges of directed graph with (1,1) (2,2) (3,3) corresponding loop.

(2) Determine whether the Relation for the directed graph shown fig 3.3 are Reflexive, symmetric, antisymmetric and transitive



Sol:- Reflexive :- The Relation is Reflexive, since the graph of the relation has a loop at every vertex

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

The Relation is not symmetric. The directed graph of the Relation has a directed edges (3,2) from 2 to 3 but there is no directed edge from 3 to 2.

Antisymmetric :- The Relation is antisymmetric
The diagraph of the Relation has at most one directed edge b/w each pair of vertices

Transitive :- The Relation is transitive the digraph of the Relation has the property whenever there are directed edges from (a,b) & (b,c), there is also a directed edge from a to c.

③ Let $X = \{1,2,3,4,5,6,7\}$ and $R = \{(x,y) / x-y$ is divisible by $3\}$ show that R is an Equivalence relation. Draw the graph of R

sol :- Reflexive :- for any $a \in X$ $a-a=0$ which is divisible by 3 $\therefore$ aRa (or) $(a,a) \in R$ for all $a \in X$ Reflexive.

(ii) Symmetric :- For any $(a,b) \in X$, if $(a-b)$ is divisible by 3 then $-(a-b)$ or $(b-a)$ is divisible by 3. That is $aRb \Rightarrow bRa$. Thus R is a symmetric

(iii) For $a, b, c \in X$ if (a,b) and $(b,c) \in R$, that is aRb and bRc, then both $(a-b)$ and $(b-c)$ are divisible by 3.

$\Rightarrow (a-c) = (a-b) + (b-c)$ which is divisible by 3

$\Rightarrow (a,c) \in R$ or aRc

Thus R is transitive

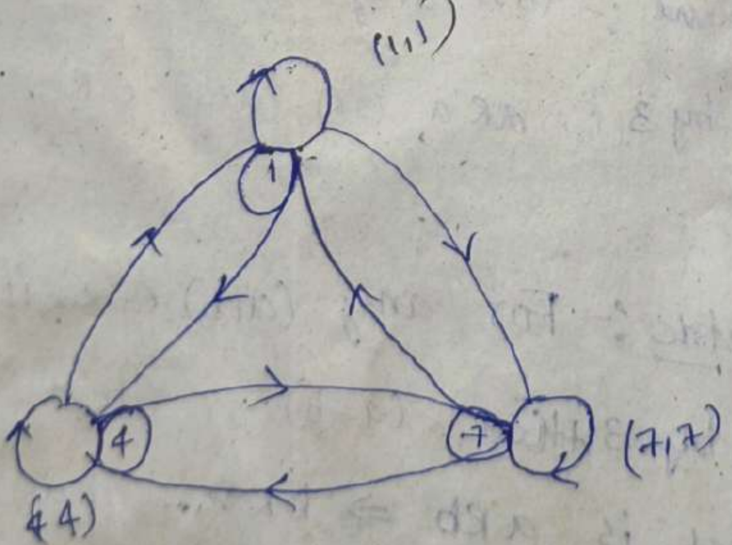
Hence, R is an equivalence

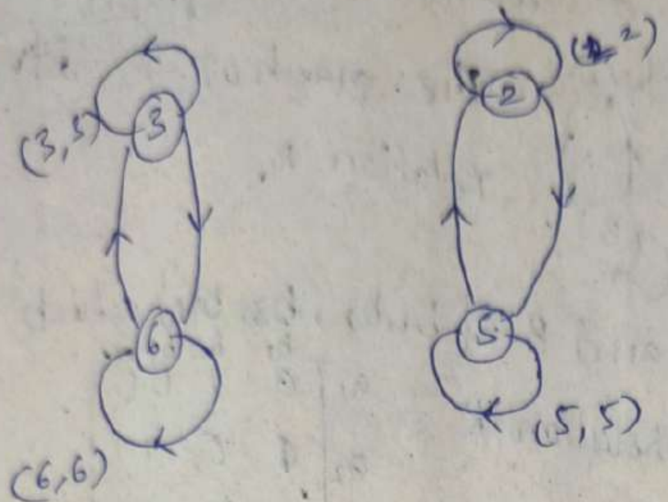
Graph of R

From the definition of given R, we note that

$R = (1,1) (1,4) (1,7) (2,2) (3,3) (3,6) (4,1) (4,4)$
$(4,7) (5,2) (5,5) (6,3) (6,6) (7,1) (7,4)$

The diagram of R is as shown below:





Relation Matrix (The matrix of a Relation)

A relation R from finite set A to finite set B can also represented by matix called the Relation Matrix of R

let $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$

$M_R = [M_{ij}]$, where

$$m_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$$

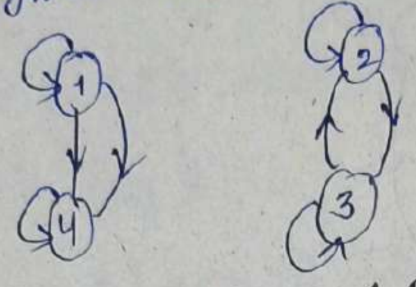
Ex :- $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(1) Let $A = \{1, 2, 3, 4\}$ be a set of Relation on the set x such that $R = \{(1,1)\ (1,4)\ (4,1)\ (4,4)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$ Draw the matrix and the graph also prove that R is Equivalence relation.

Sol :- We find the matrix of R

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

the given matrix of relation R the graph of a Relation.



The graph of the Relation R.

(2) Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$ which ordered pair are in the relation R represented by matrix

$$\begin{array}{c|c} & \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Sol:- Since R contains of those ordered paired (a_i, b_j) with $m_{ij} = 1$. It blows that

$R = \{(a_1, b_2) (a_2, b_1) (a_2 b_3) (a_2, b_4) (a_3, b_1) (a_3, b_3)\}$

<u>Composition of Relations</u>:-

Let A, B, C be sets Lit R be a relation from A to B and let S be a relation from B to C that is R is subsets of $A \times B$ and S is subsets of $B \times C$

Then the compositine relation of R and S denoted by ROS the relation consisting of ordered pair (a, c) when $a \in A$ & $c \in C$ and for union element $(a, b) \in R$ $(B, C) \in S$.

$ROS = \{(a, c) \in A \times C$ for some $b \in B$, $(a, b) \in R$ and $(b, c) \in S\}$ that is (ROS) for some $b \in B$ we have aRb & bSc

Composite of Relation :-

(1) Q:- Let R and S be the following relations on $A = \{a, b, c, d\}$ defined by

$R = \{(a,a)\ (a,c)\ (c,b)\ (c,d)\ (d,b)\}$ and

$S = \{(b,a)\ (c,c)\ (c,d),\ (d,a)\}$

Find ROS (ii) SOR (iii) ROR

Sol:- We know the operation of ROS from R and S is called compositive of relation.

1) ROS :-

$(a,a) \in R$ and $(a,a) \notin S$ therefore $(a,a) \notin ROS$

$(a,c) \in R$ and $(c,c) \in S$ $\therefore (a,c) \in ROS$

$(c,b) \in R$ and $(b,a) \in S$ $\therefore (c,a) \in ROS$

$(c,d) \in R$ and $(d,a) \in S$ $\therefore (c,a) \in ROS$

$(d,b) \in R$ ~~ROS~~ and $(b,a) \in S$ $\therefore (d,a) \in ROS$

Hence $ROS = \{(a,c)\ (c,a)\ (d,a)\}$

$R = 2^{nd}$ element

$S = 1^{st}$

(ii) SOR

$(b,a) \in S$ and $(a,a) \in R$ therefore $(b,a) \in SOR$

$(b,a) \in S$ and $(a,c) \in R$ " (b,c) SOR

$(c,c) \in S$ and $(c,b) \in R$ " $(c,b) \in SOR$

$(c,c) \in S$ and $(c,d) \in R$ " $(c,d) \in SOR$

$(c,d) \in S$ and (d,b) " $(c,b) \in SOR$

Hence SOR = {(b,c) (b,a) (c,c) (c,d) (c,b)}

(iii) ROR
(a,a) ∈ R and (a,a) ∈ R therefore (a,a) ∈ ROR
(a,a) ∈ R and (a,c) ∈ R therefore (a,c) ∈ ROR
(a,c) ∈ R and (c,b) ∈ R therefore (a,b) ∈ ROR
(a,c) ∈ R and (c,d) ∈ R therefore (a,d) ∈ ROR
(c,b) ∈ R and (c,b) ∉ R therefore (c,b) ∉ ROR
(c,d) ∈ R and (d,b) ∈ R " (c,b) ∈ ROR
(d,b) ∈ R & (d,b) ∉ R " (d,b) ∉ ROR

Closure of relations:

Let R be a relation on a set A. R may or may not have some property P, such as reflexivity, symmetry or transitivity. If we add some pairs then we have the desired property. The smallest relation S on A that contains R and posses the desired property P is called closure of R with respect to that property.

Reflexive closure: The reflexive closure $R^{(6)}$ of a relation R is the smallest reflexive relation that contains R as a closure of R can be formal by adding to R all pairs of the form (a,a) for every a ∈

which are not already in R. Thus, $R^{(r)} = R \cup I_A$

where $I_A = \{(a,a) : a \in A\}$ is the diagonal (32)

relation on A.

~~Trans~~ Symmetric closure :- The symmetric closure $R^{(s)}$ is the smallest symmetric relation that contains (y,x) if it contains (x,y), since the inverse relation R^{-1} contains (y,x) if (x,y) is in R, the symmetric closure of R is defined as

$R^{(s)} = R \cup R^{-1}$ where $R^{-1} = \{(y,x) : (x,y) \in R\}$.

Transitive closure :-

The relation obtained by adding the least number of ordered pairs to ensure transitivity is called the transitive closure of the relation R and denoted by R^+ as R^+. To make a relation R transitive One has to add all pairs of R^2, all pairs of R^3 -- unless these pairs are already in R. Thus one can calculate R^+ as the union of the form R^k.

R^* or $R^+ = R \cup R^2 \cup$ --- $\cup R^n \cup$ ---

1. R^* is transitive
2. $R \subseteq R^*$;
3. If S is any other transitive relation that contains R then $R^+ \subseteq S$

Functions

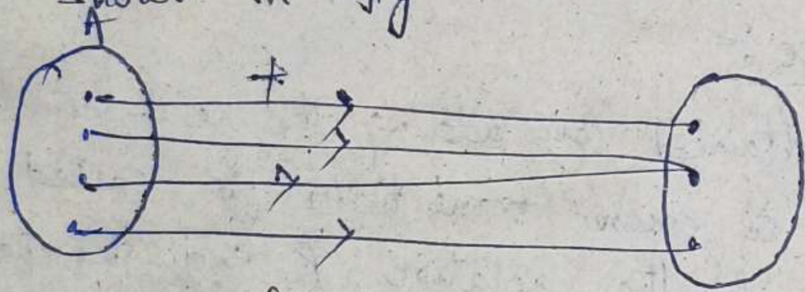
Definition:- Let A and B be two non-empty sets. A function from A to B is a set of ordered pairs

$$f \subseteq A \times B$$

with the property that for each element x in A there is a unique element y in B such that $(x, y) \in f$. The statement "f is a function from A to B" is usually represented symbolically by

$$f : A \to B \quad \text{or} \quad A \xrightarrow{f} B$$

A function can be represented pictorially as shown in fig



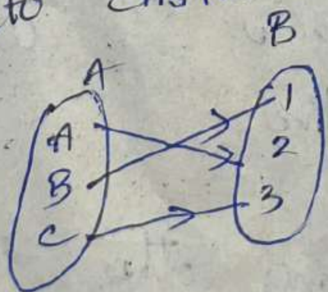
Types of functions:-

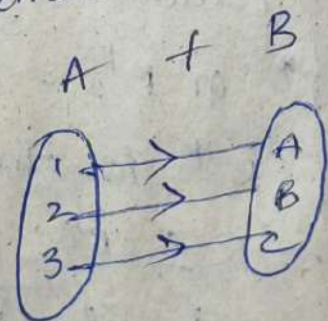
One to one function / Injective function

A function A to B is one to one or injective, if distinct element of A are mapped into distinct element of B.

$f(x_1) = f(x_2)$

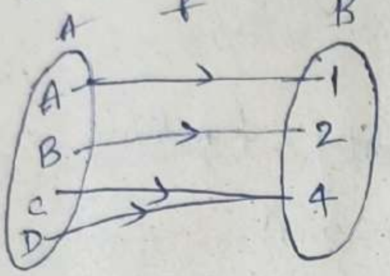
i.e $x_1 = x_2$

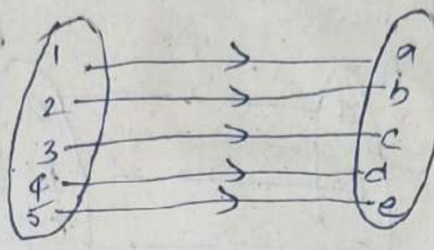




Onto function / Surjective function :-

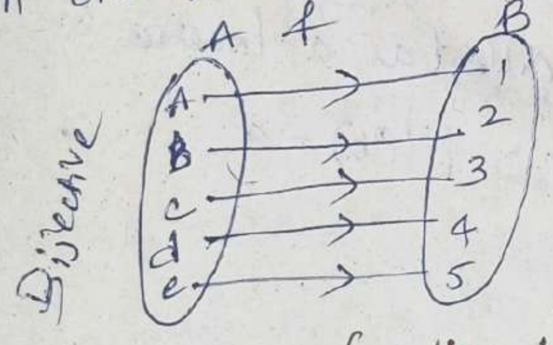
A function f from A to B in onto (or) surjective if every element of B is the Image of some element of A

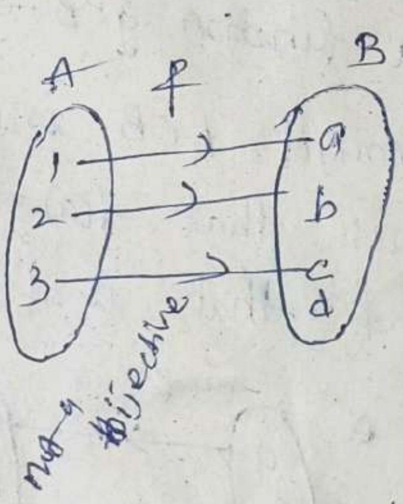




Bijective function :/ one to one & onto function :-

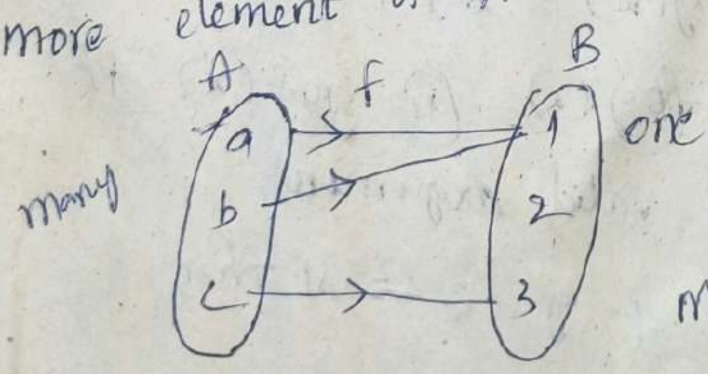
A function f from A to B is said to be bijective if f is both injuctive & surjective i.e., both one to one function





(4) Many one function :-

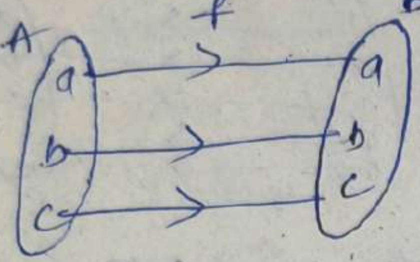
A function f from A to B is said to be many - one if and only if two (or) more element of A have same element of B



Many - one function

(5) **Indentity function :-**

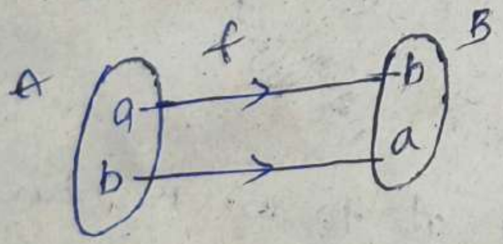
The function $f : A \rightarrow A$ defined by $f(x) = x$ for every $x \in A$ is called the Identity of A and is denoted by I_A



Identity - function
$f : A \rightarrow A$

(6) **Inverse of a function :-**

let $f : A \rightarrow B$ be a bijective function. Then a function $g : B \rightarrow A$ which is associate each element $b \in B$ is a unique element of $a \in A$ such that $f(a) = b$ is called as a Inverse of f. that is $f(a) = b \Leftrightarrow f^{-1}(b) = a$



Ex :- let the function $f : N \rightarrow N$ and $g : Z \rightarrow N$ be defined as follows, $f(x) = 3x + 2$ & $g(x) = x^2 + 1$ specific the function (i) $fog(x)$ (ii) $gof(x)$ if exist and give a valid argument.

Sol :- $f, N \longrightarrow N$ & $g : Z \rightarrow N$ then

$f(x) = 3x + 2 \qquad g(x) = x^2 + 1$

(i) $fog(x)$

$\Rightarrow 3x+2$

$\Rightarrow 3(x^2+1)+2$

$\Rightarrow 3x^2+3+2$

$fog(x) = 3x^2+5$ //

(2) $gof(x)$

x^2+1

$(3x+2)^2+1$

$9x^2+4+12x+1$

$9x^2+12x+5$ //

(2) Determine whether each of these functions is a bijective from R to R.

(i) $f(x) = 3x+4$

(ii) $f(x) = -3x^2+7$

(iii) $f(x) = \frac{x+1}{x+4}$

(iv) $f(x) = x^5+1$

Sol :- We have $f(x) = 3x+4$

one to one function

let $a_1, a_2 \in R$ and $f(a_1) = f(a_2)$

$-3a_1+4 = -3a_2+4$

$a_1 = a_2 \rightarrow$ is one to one function.

onto to function :-

let $b \in R$ any real no in R then

$f(x) = b$

$-3x+4 = b$

$x = \frac{4-b}{3} \in R$

$\frac{4-b}{3}$ is the pre-Image of b f is onto function

Since f is one to one & onto f is bijective function

(ii) We have $f(x) = -3x^2 + 7$

one to one function

let $a_1\, a_2 \in R$ and $f(a_1) = f(a_2)$

$-\cancel{3}a_1^2 + \cancel{7} = -\cancel{3}a_2^2 + \cancel{7}$

$a_1^2 = a_2^2 \Rightarrow a_1 = a_2$ f is one to one function

Onto function :-

let $b \in R$ then

$f(x) = b \Rightarrow -3x^2 + 7 = b$

$7 - b = 3x^2$

$x^2 = \frac{7-b}{3}$

$x = \left[\frac{7-b}{3}\right]^{1/2}$

$\left(\frac{7-b}{3}\right)^{1/2}$ is a pre Image of its is an onto

the given problem is one to one & onto,

Hence Bijective

(iii) We have $f(x) = \frac{x+1}{x+2}$ let A and B be the set of real no's

one to one

let $a_1\, a_2 \in R$ then $f(a_1) = f(a_2)$

$\frac{a_1+1}{a_1+2} = \frac{a_2+1}{a_2+1}$

$(a_1+1)(a_2+2) = (a_1+2)(a_2+1)$

$a_1a_2 + 2a_1 + a_2 + 2 = a_1a_2 + a_1 + 2a_2 + 2$

$2a_1 + a_2 = a_1 + 2a$

$a_1 = a_2$ f is one to one

onto function :-

let $y \in B$ then the pre image of $y \in B$ is ~~an~~ element $x \in A$

$f(x) = y \qquad \frac{x+1}{x+2} = y$

$x+1 = y(x+2)$

$x+1 = xy + 2y$

$x - xy = 2y - 1$

$x(1-y) = 2y - 1$

$x = \frac{2y-1}{1-y}$ is not a onto function

It is not a Bijective.

(iv) we have $f(x) = x^5 +$ let A & B be the set of real no's

one to one function

let a_1, a_2 such that $f(a_1) = f(a_2)$

$a_1^5 + 1 = a_2^5 + 1 = a_1 = a_2 = a_1 = a_2$

is a one to one function

onto function :-

let $y \in B$ then pre-Image of $y \in B$

is an element $x \in A$

$f(x) = y \Rightarrow x^5 + 1 = y$

$x^5 = y - 1$

$x = (y-1)^{\frac{1}{5}}$

$(y-1)^{\frac{1}{5}} \in B$ because of $y \in B$ f is onto so Bijective function.